Sample Project by Alex Beaugard -Dwyer following on his work in Chapter 13 of 'Fundamental Applied Maths'

Complete a mathematical modelling project and report based on the following brief:

"Investigate mathematically the time taken to cycle a stage of the Ras Tailteann from Listowel to Glengarriff.
Your report should include the Principle of Conservation of Energy."

Select one or more aspects of estimating the time taken. Model the problem(s) you have selected using the cycle described in The Modelling Cycle section of this document. Additional relevant material:

The Ras Tailteann is a cycle race in which the winner is an individual. But the winner is usually a member of a team who cooperate with one another during the race.

The cycle includes mountainous as well as flat terrain.

Mathematical Modelling Project

Type into the space below the title of your mathematical modelling project.

To estimate the time taken to cycle a stage of the Ras Tailteann from Listowel to Glengarriff without relying on Google to find the distance.

Word count

Do not complete the following table until after you have finished your report.

Section	**Number of words**
Introduction and Research	287
The Modelling Process	509
Interpretation of Results	98
Total	894

1. Background: I looked up websites about the Ras Tailteann. I looked at photos of Kerry to see the terrain.
2. 1st Iteration: First, I wanted to work out the length of the journey without Google Maps. I used an atlas in the school library, a ruler and the scale 1 : 25 to come up with the answer and then I then timed a friend of mine cycling 400 m to estimate the average speed. I used just these variables (**distance** and **speed**) to estimate the time.
3. 2nd Iteration: I used the parameter of **curvature** with a factor of $\frac{\pi}{2}$ because the journey is a wavy curve, not a straight line. This assumed that the road consists of joined semi-circles.
4. 3rd Iteration: I took into consideration the parameter of **slope** (i.e. the hills) and the **Principle of Conservation of Energy**. I had to use Google Maps to find a profile of the ups and downs. I learned that if you go up a hill and then down a hill, you lose out in time because you spend more time struggling up and less time freewheeling down, so the overall time increases. The Principle of Conservation of Energy helps a cyclist, but overall, the flatter the journey, the shorter the time.
5. 4th Iteration: I introduced the variables **power** and **drag forces**. I used $P = Tv$ and Drag = kv^2 to come up with findings that showed that, for example, an increase of 33% in work done increases the speed by only 10%. I used a spreadsheet to develop a table of the relationship between work and speed. I looked at how drag might be reduced by teamwork.

   Citations: Collins Atlas 1996. https//maps.google.com. www.rastailteann.com

Introduction and Research **20 marks**

The Modelling Process	**50 marks**

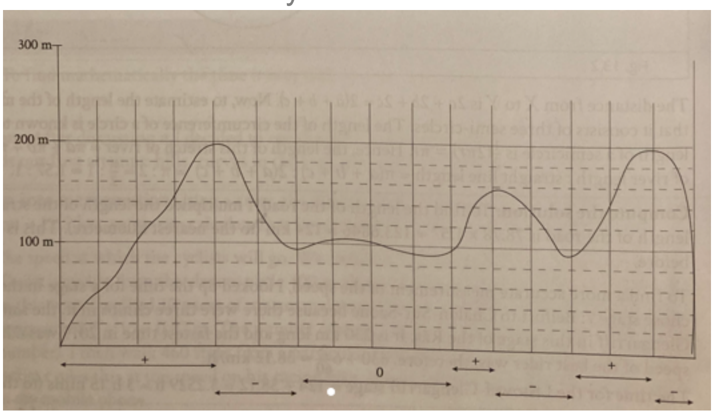
1st Iteration: I decided to estimate the time taken as though the journey was a straight line. Using a ruler, the atlas, and the ratio 1 : 25, the distance from Listowel to Glengarriff was estimated to be 78.78 km. Using my cyclist friend cycling at full speed over 400 m. I got the speed to be 38 km/h.
The estimated time was, therefore, 78.78/38 = 2 h 04 mins.

2nd Iteration: Using the parameter of the curvature of the road. I assumed that the journey was not a straight line but a series of semi-circles.

The ratio of the lengths of the curve to the diameter is $\pi r : 2r = \pi : 2 = \frac{\pi}{2} : 1 = 1.57 : 1$
This gives the estimated length of the journey as a more accurate 78.78 x 1.57 = 124 km. The estimated time to do the journey is now 124/38 = 3 h 15 m. This seems a much better answer.

3rd Iteration: I introduced the further parameters: height and energy – especially the Principle of Conservation of Energy (PoCoE). As (s)he cycles up a hill, the cyclist uses energy which leaves them at the top of the hill (with potential energy). Now, going downhill, the cyclist turns the PE into KE and gains speed. It might appear that the PoCoE means that you will gain on the way up and lose on the way down – so the time will be the same as if the terrain was flat. This is nonsense. You lose more because you spend a lot more time struggling up than zooming down. So the PoCoE helps, but energy is lost overall.

I estimated speed up a hill and down a hill. I divided the journey into 16ths. The road climbs for 7/16 of the way, falls for 5/16 and is flat for 4/16. The respective speeds were 63, 13 and 38 km/h. The time for the journey came to 5 h 36 mins. This is my most accurate estimation yet.



4th Iteration: I introduced drag. If Drag = kv^2 and if $P = Tv$, and if the cyclist is travelling at a constant speed, then $T = kv^2$ and hence $P = kv^3$. I assumed the cyclist was going at 40 km/h. $P = k(40)^3 = 64000k$ watts. I then looked at the percentage increase in power that the cyclist needed in order to increase the speed by various amounts.

TABLE:

New speed	**% increase in speed**	**New power output**	**% increase in power**
41	2.5%	68921k	7.689%
42	5.0%	74088k	15.763%
43	7.5%	79507k	24.230%
44	10%	85184k	33.1%
45	12.5%	91125k	43.38%
46	15%	97336k	52.1%
47	17.5%	68921k	62.22%
48	20%	103823k	72.8%
49	22.5%	117649k	83.83%
50	25%	125000k	95.31%

To increase the speed by 25% requires an increase in power of 93.31%. From this, I learnt that it is desperately important for a participant to reduce drag by

- moving in a line as a team
- distributing the power amongst the group
- design of bike and equipment to reduce drag.

If this is so, the estimated time of 5 h 36 min still stands.

Interpretation of Results 15 marks

My key findings are as follows: the Principle of Conservation of Energy means you should get back the energy that you expend. This is not quite true. You lose more time going uphill than you gain going down.

Also, energy is lost due to drag forces. These can waste a lot of energy. If the cyclist can have well designed, streamlined equipment, can work as part of a team, this will reduce waste of energy.

I concluded that the estimated time to complete the journey is 5 h 36 mins, if the cyclist is part of a team.