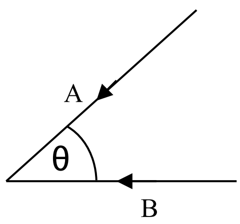
1986 Q1

(a) A particle with speed 150m/s begins to decelerate uniformly at a certain instant while another particle starts from rest 8s later and accelerates uniformly. When the second particle had travelled 135m both particles have a speed of 30m/s.
(i) Show the motion of both particles on the same speed-time graph.
(ii) How many seconds after the commencement of deceleration does the first particle come to rest? **(21.25s)**

(b) A particle starting from rest at p moves in a straight line to q with uniform acceleration. In the first second it travels 5m. In the last three seconds of its motion before reaching q it travels $\frac{9}{25}$ of $|pq|$.
Find the time in seconds from p to q. **(15s)**

1986 Q2

Two straight roads intersect at an angle θ, tan $\theta = {}^4/_3$. Cars A and B move towards the point of intersection at 16m/s and v m/s, respectively. If the magnitude of the velocity of A relative to B is 16m/s, find v.
If at a given instant A is 96m and B is 38.4m from the intersection, calculate
(i) the shortest distance between them in their subsequent motion
(ii) the distance, to the nearest metre, between the two cars 2 seconds before the instant when are nearest to each other



1986 Q3

A particle is projected with speed u at an angle α to the horizontal. If the maximum height reached is the same as the total horizontal range, show that tan $\alpha = 4$

The particle moves at right angles to its original direction of motion after a time t_1 and then strikes the horizontal plane after 8s, both times measured from the instant of projection.

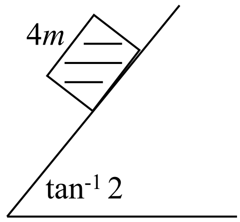
Show $u = g\sqrt{17}$
Calculate t_1.

1986 Q4

A smooth particle of mass $4m$ rests on the smooth inclined face of a wedge of mass m and slope $\tan^{-1} 2$. The wedge is free to move on a rough horizontal table, the coefficient of friction being $^1/_3$. When the system is released from rest, the wedge moves with acceleration p parallel to the table.

(i) Show on separate diagrams the forces acting on the wedge and on the particle.

(ii) Calculate p on terms of g.



1986 Q5

(a) A smooth sphere P of mass $3m$ and velocity $4u$ impinges directly on a smooth sphere Q of mass $5m$ and velocity $2u$, moving in the same direction. The coefficient of restitution is e.

(i) For what value of e will the velocity of P be halved by the impact?

(ii) Show that whatever the value of e in $0 < e < 1$, the velocity of Q after impact exceeds $2u$. **(2.75u?)**

(b) Two smooth spheres of masses 4 kg and 2 kg impinge obliquely. The 2 kg mass is brought to rest by the impact.

(i) Prove that, before impact, they were moving in directions perpendicular to each other.

(ii) Show that, as a result of impact, the kinetic energy gained by the 4 kg mass is equal to half that lost by the 2 kg mass.

1986 Q6

(a) A particle moves with simple harmonic motion through two points p and q 1.2m apart. Its velocity is the same at p as at q. It takes 3seconds to move from p to q and 3 seconds to move from q to p ie passing q the next time.
Using a diagram, or otherwise find the period of the particle and the amplitude.

(b) An elastic string of natural length $2a$ is stretched between two points which are $2\sqrt{3}\,a$ apart. A particle of weight W is attached to its midpoint and hangs in equilibrium with the string inclined at an angle of 30^0 to the horizontal.
Show that the modulus of elasticity of the string is W. *($^W/_a$)*

1986 Q7

State and prove the relationship between the coefficient of friction, μ and the angle of friction.
One end of a uniform rod rests on a rough horizontal floor and the other end rests in contact with a rough vertical wall, the coefficient of friction in both cases being μ.
The rod makes an acute angle, θ, with the wall.

(i) Show on a diagram the forces acting on the rod.

(ii) Show that when the rod is on the point of slipping $\tan\theta = \dfrac{2\mu}{1-\mu^2}$

1986 Q8

(a) Prove that the moment of inertia of a uniform circular disc, of mass *m*, and radius *r*, about an axis through its centre perpendicular to its plane is ½ mr^2.

(b) A uniform circular disc, starts from rest and rolls down a plane of inclination 30^0. The plane, being rough, prevents sliding. Using the principle of conservation of energy, or otherwise, show that the uniform acceleration of the disc is $^g/_3$ m/s^2.

1986 Q9

(a) A square plate is immersed vertically in water with an edge of length *d* in the surface of the water. Find how far below the surface is the horizontal line which divides the square into two rectangles on each of which the thrust is the same.

(b) A hollow spherical shell of external diameter 2m and uniform thickness 0.5m, floats in a liquid with half of its volume immersed. If the relative density of the liquid is 1.4, find the relative density of the material of the shell.

1986 Q10

(a) Solve the differential equation $x\dfrac{dy}{dx} = y(1-x)$ if $y = 3$ when $x = 1$.

(b) A particle moves in a straight line in a medium whose resistance is proportional to the cube of its speed. No other force acts on the body. The speed falls from 15m/s to 7.5m/s in a time of *t* seconds.
Show that the distance travelled in this time is 10*t* m.

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