1985 Q1

A bus 12.5m long travels with constant acceleration. The front of the bus passes a point, p, with speed u while the rear of the buss passes p with speed v.
Find in terms of u and v

(i) the time taken by the bus to pass p. ($t = \frac{25}{u+v}$)

(ii) what fraction of the length of the bus passes the point p in half this time. ($\frac{3u+v}{4(u+v)}$)

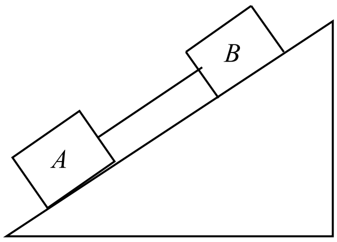
A particle is projected from a point o, with initial velocity u, up a plane inclined at an angle of 60^0 to the horizontal. The direction of projection makes an angle θ with the inclined plane. (The plane of projection is vertical and contains the line of greatest slope) the maximum height reached above the inclined plane is H. Express

(i) the velocity and displacement from o of the particle after t seconds, in terms of $\vec{i}$ and $\vec{j}$, where these are the unit vectors along and perpendicular to the plane, respectively.

(ii) H, in terms of u and θ

(iii) the time interval, in terms of $\sin 2\theta$, between the instants when the particle is at a height, $H \sin^2 \theta$ above the inclined plane.

Two blocks A and B have masses 2 kg and x kg respectively.
They are connected by a string and slide down an inclined plane which makes an angle $\sin^{-1}(3/5)$ with the horizontal.
The coefficient of friction between A and the plane is ¼ and between B and the plane is ½ .



(i) Show on a diagram the forces acting on each block when the system is released from rest.

(ii) Find the acceleration a of the system in terms of x.

(iii) For what value of x would the acceleration of the blocks be $0.9a$?

State the laws governing the oblique collision of elastic spheres.
A smooth sphere A impinges obliquely on an identical smooth sphere B which is at rest. The direction of A before and after impact makes an angle 60^0 and θ, respectively, with the line of centres.

(i) Prove that $\tan\theta = \frac{2\sqrt{3}}{1-e}$ where e is the coefficient of restitution between the spheres.

(ii) Show that the maximum percentage loss in kinetic energy due to the impact is 12½ %

(iii) For what value of e will the kinetic energies of A and B after impact be in the ratio 7:1 ?

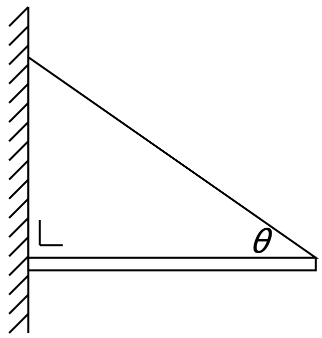
(a) Two cars A and B are moving along straight roads which are at right angles to each other, with uniform velocities 3m/s and 4 m/s, respectively. When B is at the crossroads, A is 100m away. Calculate the time interval for which the distance between the cars is not greater than 82m.

(b) A car of mass 750kg attains a maximum speed of 30m/s when travelling down an incline of 1 in 25 with the engine switched off. It can attain a maximum speed of 20m/s up the same incline when the engine is working. The resistance to motion in each case is proportional to the square of the speed. Find
(i) the power at which the engine is working
(ii) the maximum speed of the car along a level road, if it works at the same power and its resistance is again proportional to the square of the speed.

Define simple harmonic motion.
A body of mass 0.25kg hangs from a spiral spring. When pulled down 10cm below its equilibrium position and released, it vibrates with simple harmonic motion of period 2 seconds.

(i) Find its velocity as it passes through the equilibrium position.
(ii) What is the shortest time taken to travel from a point 2 cm below the position to a point 2 cm above the equilibrium position?
(iii) Find the elastic constant of the spring.
(iv) By how much will the spring shorten when the body is removed?

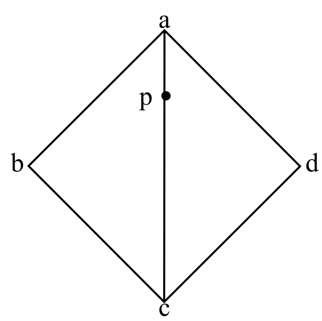
One end of a uniform metre stick of mass m is placed against a vertical wall. The other end is held by a light inelastic string making an angle θ with the metre stick. The coefficient of friction μ, between the end of the metre stick and the wall is 0.4



(i) Show in a diagram the forces acting on the metre stick.
(ii) Show that if the metre stick is to remain in equilibrium the maximum value of θ is given by $\tan\theta = \mu$.
(iii) If a mass m is suspended from the metre stick at a distance x from the wall, show that the stick is on the point of slipping when

$$\tan\theta = \frac{2(1+2x)}{5(3-2x)}$$

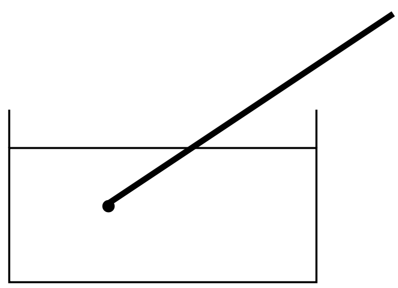
A uniform square lamina $abcd$, of mass $3m$ and side $\sqrt{2}$, is free to rotate with its plane vertical about a smooth horizontal axis through a point p on the line ac. A mass m is attached at each of the points a and c.



(i) If $|ap| = 1 - x$, prove that the moment of inertia of the system about a horizontal axis through p is $m(3 + 5x^2)$.
(ii) If the system oscillates about p, find in terms of x, the period for small oscillations.
(iii) Find the value of x which gives the minimum period when oscillations are small.

(a) A piece of gold-aluminium alloy of mass 10kg weighs 72N in water. If the relative densities of gold and aluminium are 19.6 and 2.45 respectively, find
(i) the relative density of the alloy.
(ii) the mass of each metal in the alloy
(iii) what fraction of the total volume of the alloy is gold.

(b) A uniform rod of relative density 0.25 is free to turn about its lower end, which is fixed at a depth 0.4m in water. The rod is in equilibrium when partially immersed and making an angle of 60^0 with the vertical. Find the length of the rod.



(a) Find the solution of the differential equation $3y^2(x-1)\frac{dy}{dx} = 1 + y^3$ if y = 0 when x = 2.

(b) A particle of mass *m* moves in a straight line. The only force acting on it being a resistance mkv^2, where *v* is its speed and *k* is a constant. It is initially projected from the point *o* with speed *u*. When the particle reaches a point *p* on the line its speed is $^u/_3$. (i) Show that the average speed between *o* and *p* is ½ *u*log3.

(ii) Find the speed of the particle when it is at the midpoint of [*op*].